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SYNCHRONOUS OPERATION OF STANDBY UNITS WITH PROVISION OF MAINTENANCE FACILITY

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Abstract

The aim of this paper is to analyze the system consisting of one main unit and three cold standby units. The system is provided with maintenance facility under the constraint of synchronous operation of all the cold standby units. This provision of maintenance has a significant role in order to improve the reliability of the whole system. There is a single repairman available for both purposes: repair as well as maintenance. System effectiveness measures such as behavior of Mean time to system failure (MTSF), Availability, Busy period and Cost benefit analysis has been done for the present study using Semi-markov process and Regenerative point technique. Graphical interpretation has also been performed considering particular cases.

Keywords:

Standby systems;
Semi-markov process;
Regenerative point technique.

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1. Introduction:

Reliability is the probability that the item operates a specified task under stated environmental condition in a given specified time. Under the field of reliability engineering, Maintainability holds a vital role to keep the system work smoothly and efficiently. Practically, it is impossible that any system operates in a fail-free manner throughout its lifetime. In such situations, the systems are repaired whenever the termination occurs. This brings into account the concept of Maintainability and Repairability. Various researchers have done a tremendous work considering provision of maintenance facility for the system. Numerous reliability models have been studied for different work mechanism of the systems. But no study has been performed with synchronous working of cold standby units along with provision of maintenance facility. The present study deals with such situation.

Practically, we came across a situation while studying different innovative industrial standby systems. The power plant working in Bunge Pvt. Ltd. situated at Rajpura, Punjab

(India) drove our interest to do its reliability analysis. Earlier, the plant consists of three low pressure boilers. But with the passage of time in advancement of technology in engineering systems, the new system was introduced which comprised one main high pressure boiler and old three low pressure boilers were made standby units. Our study is based upon new system which is presently working there.

The system consists of one main unit and three cold standby units. The maintenance facility is provided in the system to ensure the proper functioning of system before it comes across any failure. Also, all the standby units become operative in case of failure in main unit; because the capacity of three standby units to perform operation of the system is equivalent to that of single main unit. Therefore, there is a synchronous working of standby units. There is only one repairman available for performing repair as well as maintenance operations. At a time, all standby units cannot fail simultaneously, i.e., failure cannot occur in any of the two among three cold standby units in a single state. Also, all the standby units go together under maintenance as they are connected in series mode. On failure of one cold standby unit, all other standby units go to standby state. Repair and Maintenance is done on First-cum-First-Serve (FCFS) basis.

2. Notations:

λ	Constant failure rate of main unit (Unit 1)
$\lambda_1/ \lambda_2/ \lambda_3$	Constant failure rate of cold standby units (Unit 2/3/4)
α_1	Constant rate of Unit 1 (main unit) to go under maintenance
α_2	Constant rate of Unit 2,3 and 4 (all of the three standby units) to go under maintenance
$g(t)/ G(t)$	pdf/ cdf of repair time of the main unit at failed state (Unit 1)
$g_1(t)/ G_1(t)$	pdf/ cdf of repair time of the standby unit at failed state (Unit 2)
$g_2(t)/ G_2(t)$	pdf/ cdf of repair time of the standby unit at failed state (Unit 3)
$g_3(t)/ G_3(t)$	pdf/ cdf of repair time of the standby unit at failed state (Unit 4)
$h_1(t)/ H_1(t)$	pdf/ cdf of maintenance time of the main unit (Unit 1)
$h_2(t)/ H_2(t)$	pdf/ cdf of maintenance time of standby units all together (Unit 2, 3 and 4)
$O_I/ O_{II}/ O_{III}/ O_{IV}$	Unit 1/2/3/4 is in operative state
$CS_{II}/ CS_{III}/ CS_{IV}$	Unit 2/3/4 is in cold standby state
$U_{mI}/U_{mII}/U_{mIII}/U_{mIV}$	Unit 1/2/3/4 is under maintenance respectively
$U_{MI}/U_{MII}/U_{MIII}/U_{MIV}$	Unit 1/2/3/4 is under maintenance respectively from the previous state, i.e., maintenance is continuing from previous state
$F_{rI}/F_{rII}/F_{rIII}/F_{rIV}$	Unit 1/2/3/4 is under repair respectively
$F_{wrI}/F_{wrII}/F_{wrIII}/F_{wrIV}$	Unit 1/2/3/4 is waiting for repair respectively
$F_{RI}/F_{RII}/F_{RIII}/F_{RIV}$	Unit 1/2/3/4 is under repair respectively from the previous state, i.e., repair is continuing from previous state

3. Transition probabilities and mean sojourn times:

A state transition diagram in fig. 1 shows various transitions of the system. The epochs of entry into states 0, 1, 2, 3, 11, 12 and 13 are regenerative points and thus these are regenerative states. The states 4, 5, 6, 7, 8, 9 and 10 are failed states.

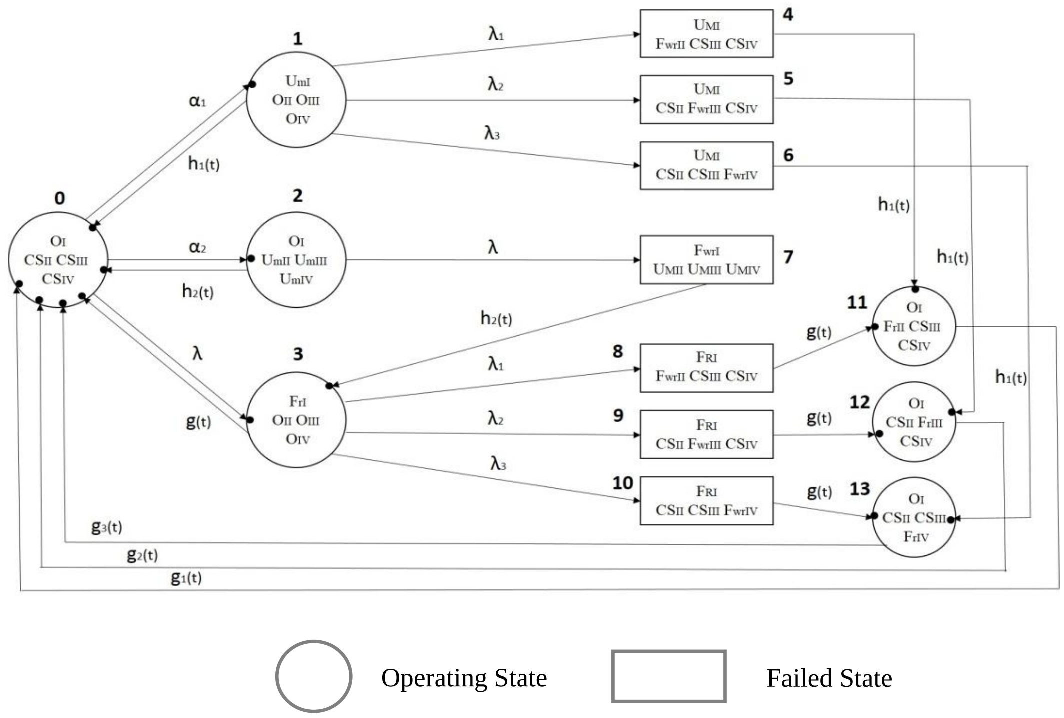

Fig. 1

The non-zero elements p_{ij}, are obtained as under:

$$p_{01} = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \lambda} \qquad p_{02} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \lambda}$$

$$p_{03} = \frac{\lambda}{\alpha_1 + \alpha_2 + \lambda} \qquad p_{10} = h_1^*(\lambda_1 + \lambda_2 + \lambda_3)$$

$$p_{14} = \frac{\lambda_1[1 - h_1^*(\lambda_1 + \lambda_2 + \lambda_3)]}{\lambda_1 + \lambda_2 + \lambda_3} = p_{1,11}^{(4)} \qquad p_{15} = \frac{\lambda_2[1 - h_1^*(\lambda_1 + \lambda_2 + \lambda_3)]}{\lambda_1 + \lambda_2 + \lambda_3} = p_{1,12}^{(5)}$$

$$p_{16} = \frac{\lambda_3[1 - h_1^*(\lambda_1 + \lambda_2 + \lambda_3)]}{\lambda_1 + \lambda_2 + \lambda_3} = p_{1,13}^{(6)} \qquad p_{20} = h_2^*(\lambda)$$

$$p_{27} = 1 - h_2^*(\lambda) = p_{23}^{(7)} \qquad p_{30} = g^*(\lambda_1 + \lambda_2 + \lambda_3)$$

$$p_{38} = \frac{\lambda_1[1 - g^*(\lambda_1 + \lambda_2 + \lambda_3)]}{\lambda_1 + \lambda_2 + \lambda_3} = p_{3,11}^{(8)} \qquad p_{39} = \frac{\lambda_2[1 - g^*(\lambda_1 + \lambda_2 + \lambda_3)]}{\lambda_1 + \lambda_2 + \lambda_3} = p_{3,12}^{(9)}$$

$$p_{3,10} = \frac{\lambda_3[1 - g^*(\lambda_1 + \lambda_2 + \lambda_3)]}{\lambda_1 + \lambda_2 + \lambda_3} = p_{3,13}^{(10)} \qquad p_{4,11} = h_1^*(0) = p_{5,12} = p_{6,13}$$

$$p_{73} = h_2^*(0) \qquad p_{8,11} = g^*(0) = p_{9,12} = p_{10,13}$$

$$p_{11,0} = g_1^*(0) \qquad p_{12,0} = g_2^*(0)$$

$$p_{13,0} = g_3^*(0)$$

By these transition probabilities, it can be verified that

$p_{01}+p_{02}+p_{03}=1$

$p_{10}+p_{14}+p_{15}+p_{16}=1$ $\quad$ $p_{10}+p_{1,11}^{(4)}+p_{1,12}^{(5)}+p_{1,13}^{(6)}=1$

$p_{20}+p_{27}=1$ $\quad$ $p_{20}+p_{23}^{(7)}=1$

$p_{30}+p_{38}+p_{39}+p_{3,10}=1$ $\quad$ $p_{30}+p_{3,11}^{(8)}+p_{3,12}^{(9)}+p_{3,13}^{(10)}=1$

$p_{40}+p_{41}+p_{4,10}=1$ $\quad$ $p_{40}+p_{41}+p_{42}^{(10)}=1$

$p_{4,11}=p_{5,12}=p_{6,13}=1$ $\quad$ $p_{73}=1$

$p_{8,11}=1=p_{9,12}=p_{10,13}$ $\quad$ $p_{11,0}=p_{12,0}=p_{13,0}=1$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as –

$$m_{ij}=\int_0^\infty t\,dQ_{ij}(t)=-q_{ij}^{*'}(0), Thus-$$

$m_{01}+m_{02}+m_{03}=\mu_0$

$m_{10}+m_{14}+m_{15}+m_{16}=\mu_1$ $\quad$ $m_{10}+m_{1,11}^{(4)}+m_{1,12}^{(5)}+m_{1,13}^{(6)}=m_1$

$m_{20}+m_{27}=\mu_2$ $\quad$ $m_{20}+m_{23}^{(7)}=m_2$

$m_{30}+m_{38}+m_{39}+m_{3,10}=\mu_3$ $\quad$ $m_{30}+m_{3,11}^{(8)}+m_{3,12}^{(9)}+m_{3,13}^{(10)}=k$

where,

$k=\int_0^\infty \overline{G}(t)dt$ $\quad$ $m_1=\int_0^\infty \overline{H}_1(t)dt$

$m_2=\int_0^\infty \overline{H}_2(t)dt$

The mean sojourn time in the regenerative state i (μ_i) is defined as the time of stay in that state before transition to any other state, then we have -

$$\mu_0=\frac{1}{\lambda+\alpha_1+\alpha_2} \qquad \mu_1=\frac{1-h_1^*(\lambda_1+\lambda_2+\lambda_3)}{\lambda_1+\lambda_2+\lambda_3}$$

$$\mu_2=\frac{1-h_2^*(\lambda)}{\lambda} \qquad \mu_3=\frac{1-g^*(\lambda_1+\lambda_2+\lambda_3)}{\lambda_1+\lambda_2+\lambda_3}$$

$\mu_4=-h_1^*(0)=\mu_5=\mu_6$ $\quad$ $\mu_7=-h_2^*(0)$

$\mu_8=-g^*(0)=\mu_9=\mu_{10}$ $\quad$ $\mu_{11}=-g_1^*(0)$

$\mu_{12}=-g_2^*(0)$ $\quad$ $\mu_{13}=-g_3^*(0)$

4. Mean time to system failure:

The mean time to system failure when the system starts from the state 0, is

$$T_0=\frac{N}{D}$$

where

$$N = \mu_0 + \mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03}$$
$$D = 1 - p_{10} p_{10} - p_{02} p_{20} - p_{03} p_{30}$$

5. Expected up-time of the system:

The steady state availability of the system is given by

$$A_0 = \frac{N_1}{D_1}$$

where

$$N_1 = \mu_0 + \mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03} + k_1[p_{01} p_{1,11}^{(4)} + p_{3,11}^{(8)}(p_{03} + p_{02} p_{23}^{(7)})]$$
$$+k_2[p_{01} p_{1,12}^{(5)} + p_{3,12}^{(9)}(p_{03} + p_{02} p_{23}^{(7)})] + k_3[p_{01} p_{1,13}^{(6)} + p_{3,13}^{(10)}(p_{03} + p_{02} p_{23}^{(7)})]$$

$$D_1 = \mu_0 + m_1 p_{01} + m_2 p_{02} + k[p_{03}(p_{10} + p_{02} p_{23}^{(7)})] + k_1[p_{01} p_{1,11}^{(4)} + p_{3,11}^{(8)}(p_{03} + p_{02} p_{23}^{(7)})]$$
$$+k_2[p_{01} p_{1,12}^{(5)} + p_{3,12}^{(9)}(p_{03} + p_{02} p_{23}^{(7)})] + k_3[p_{01} p_{1,13}^{(6)} + p_{3,13}^{(10)}(p_{03} + p_{02} p_{23}^{(7)})]$$

6. Busy period of a repairman (Repair Only):

The steady state busy period of the system is given by:

$$B_R = \frac{N_2}{D_1}$$

where

$$N_2 = W_3[p_{03} + p_{02} p_{23}^{(7)}] + W_{11}[p_{01} p_{1,11}^{(4)} + p_{3,11}^{(8)}(p_{03} + p_{02} p_{23}^{(7)})]$$
$$+W_{12}[p_{01} p_{1,12}^{(5)} + p_{3,12}^{(9)}(p_{03} + p_{02} p_{23}^{(7)})] + W_{13}[p_{01} p_{1,13}^{(6)} + p_{3,13}^{(10)}(p_{03} + p_{02} p_{23}^{(7)})]$$

and D_1 is already specified.

7. Busy period of a repairman (Maintenance Only):

The steady state busy period of the system is given by:

$$B_M = \frac{N_3}{D_1}$$

where

$$N_3 = W_1 p_{01} + W_2 p_{02}$$

and D_1 is already specified.

8. Expected no. of visits of repairman:

The steady state expected no. of visits of the repairman is given by:

$$V_R = \frac{N_4}{D_1}$$

where

$N_4 = p_{01} + p_{02} + p_{03} = 1$

and D_1 is already specified.

9. Profit Analysis:

The expected profit incurred of the system is -

$$P = C_0 A_0 - C_1 B_R - C_2 B_M - C_3 V_R$$

C_0 = Revenue per unit up time of the system
C_1 = Cost per unit up time for which the repairman is busy in repair
C_2 = Cost per unit up time for which the repairman is busy doing maintenance
C_3 = Cost per visit of the repairman

10. Graphical interpretation and conclusion:

For graphical analysis following particular cases are considered:

$g(t) = \beta e^{-\beta t}$ $\qquad g_1(t) = \beta_1 e^{-\beta_1 t}$

$g_2(t) = \beta_2 e^{-\beta_2 t}$ $\qquad g_3(t) = \beta_3 e^{-\beta_3 t}$

$h_1(t) = \gamma_1 e^{-\gamma_1 t}$ $\qquad h_2(t) = \gamma_2 e^{-\gamma_2 t}$

Graphical study has been made for the MTSF and the profit with respect to failure rate of main unit (λ), revenue per unit uptime of the system (C_0) for different values of rate of failure rate of main unit (λ) and cost of repairman for busy in doing maintenance (C_2) for different values of rate of failure rate of main unit (λ).

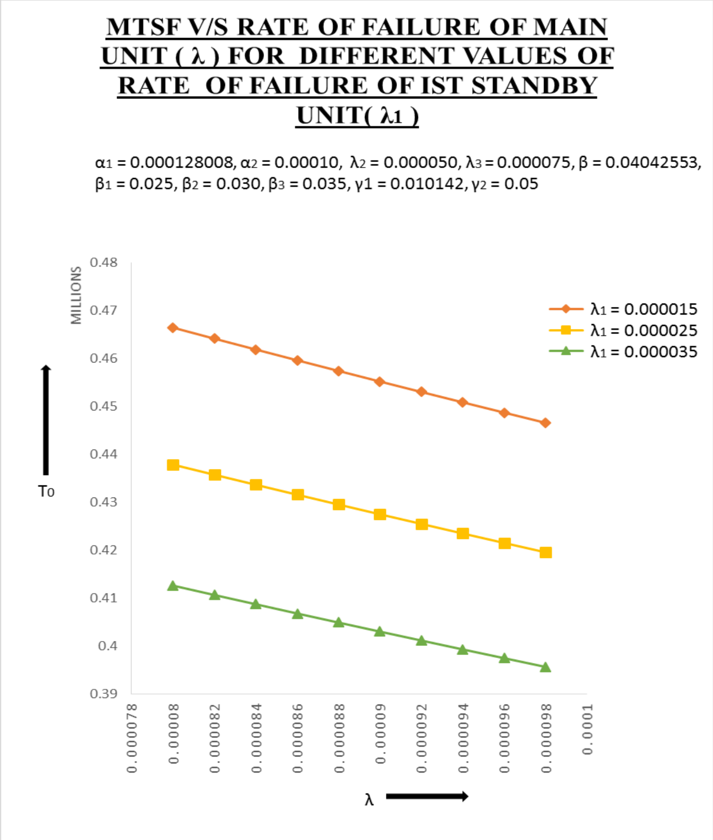

Fig. 2

Fig. 2 shows the behaviour of MTSF w.r.t. failure rate of main unit (λ) for different values of rate of failure of Ist standby unit (λ_1). It is clear from the graph that MTSF gets decreased with the increase in the values of the failure rate of main unit (λ). Also, the MTSF decreases as failure rate of Ist standby unit (λ_1) increases.

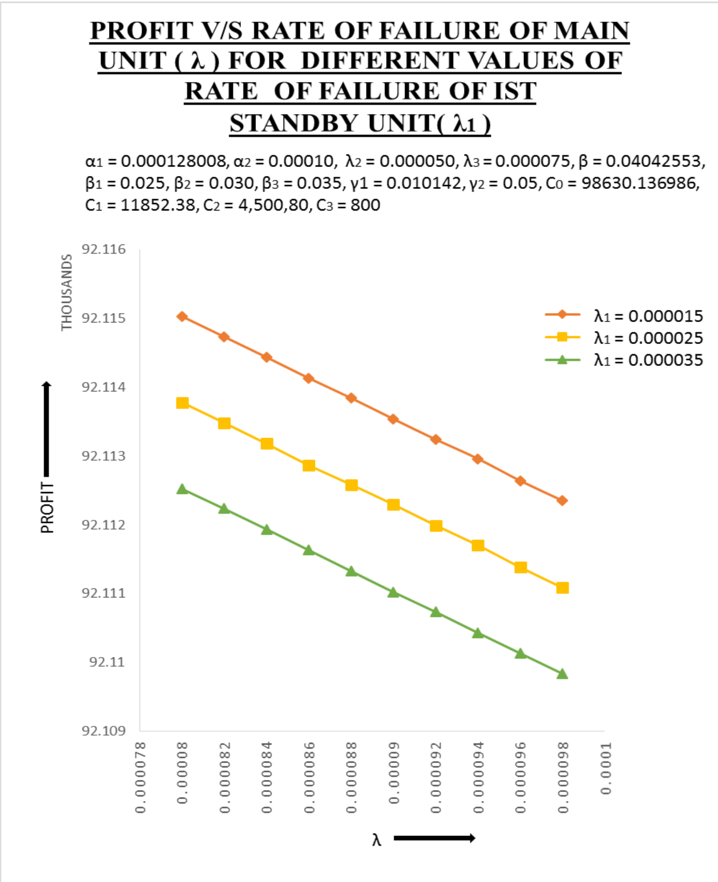

Fig. 3

Fig. 3 interprets the behaviour of profit w.r.t. to failure rate of main unit (λ) for different values of failure rate of Ist standby unit (λ_1). As the values of failure rate of main unit (λ) increases, the profit decreases. Also, the profit decreases as failure rate of Ist standby unit (λ_1) increases.

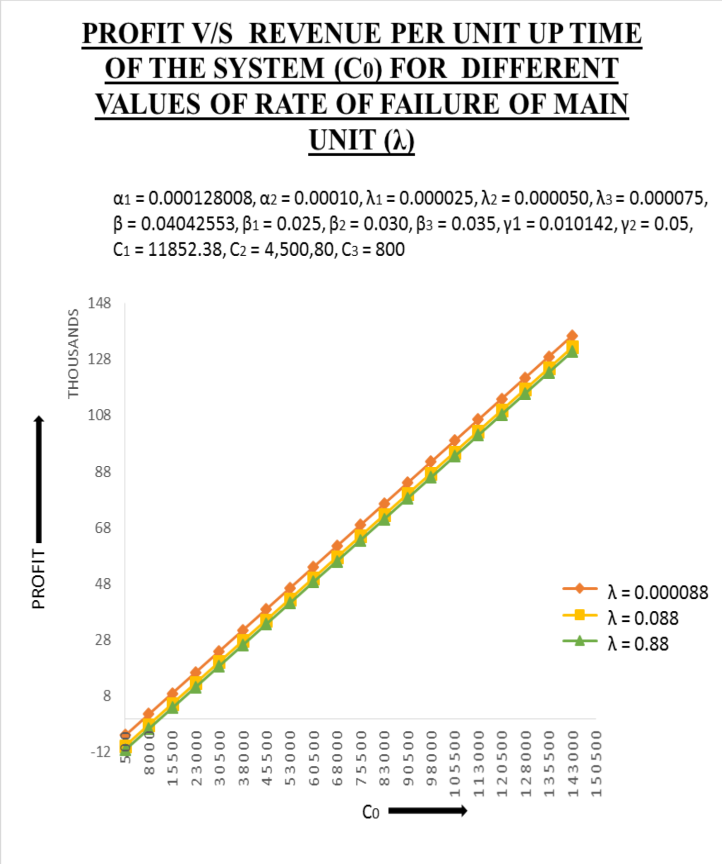

Fig. 4

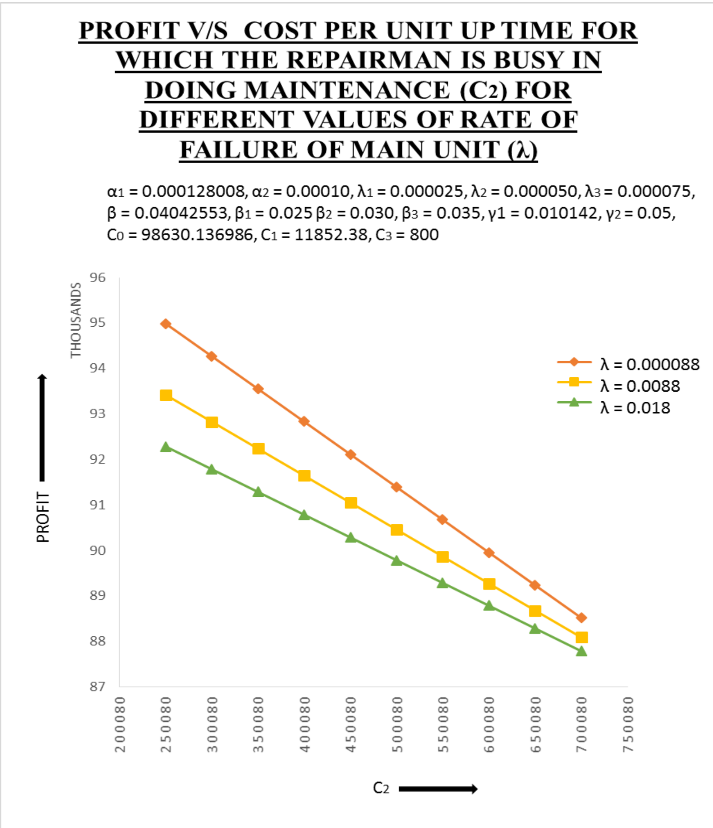

Fig. 5

Fig. 4 depicts the behaviour of the profit w.r.t. revenue per unit uptime of the system (C_0) for different values of rate of failure of main unit (λ). It can be interpreted that the profit increases with increase in the values of C_0. Following conclusions can be drawn from the graph:

1. For λ = 0.000088, profit is > or = or < according as C_0 > or = or < 6498.20, i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 6498.20 to get positive profit.
2. For λ = 0.088, profit is > or = or < according as C_0 > or = or < 10175, i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 10175 to get positive profit.
3. For λ = 0.88, profit is > or = or < according as C_0 > or = or < 11642, i.e., i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 11642 to get positive profit.

Fig. 5 shows the behaviour of profit w.r.t. to Cost per unit uptime for which the repairman is busy doing Maintenance (C_2) for different values of rate of failure of main unit (λ). As the value of Cost per unit uptime for which the repairman is busy in Maintenance (C_2) increases, the profit decreases. Also, the profit decreases as failure of main unit (λ) increases.

11. Conclusion:

It is concluded from the present study that the Mean time to system failure and Profit gets decreased as the failure rate increases. The cut-off points obtained from graphical interpretation provide the assistance to determine proper upper/lower acceptable values of rates/costs so that the economy of the company or a firm continues to have financial gain. The maintenance activities are very crucial to improve availability and reliability of a system and its performance at a minimum cost. With the help of this analysis and graphical interpretation, system effectiveness measures can be obtained so that any firm using such system can build a proper model to improve its functioning.

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